

Raising the Floor and Lifting The Ceiling: Math For All

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“Math. The bane of my existence for as many years as I can count. I cannot relate it to my life or become interested in what I’m learning. I find it boring and cannot find any way to apply myself to it since I rarely understand it.” (The Alberta Teachers’ Association, 2003, p.28)

Perhaps more than any other discipline, the teaching of mathematics lends itself to procedural recipes where students memorize and duplicate procedures by rote: if it looks like this, do that to it. “If one believes that mathematics is mostly a set of procedures—rules and truths—and the goal is to help students become proficient executors of the procedures, then it is understandable that mathematics would be learned best by mastering the material incrementally, piece by piece” (Stigler and Hiebert, 1999, p.90). Teaching practices that commonly flow from this view are demonstration, repetition and individual practice. In addition to being a misunderstanding of the discipline of mathematics itself, this belief also colors people’s views about who can learn mathematics. Curricula and teaching practices are often based on what Mighton calls a destructive ignorance “that leads us, even in this affluent age, to neglect the majority of children by educating them in schools in which only a small minority are expected to naturally love or excel at learning” (2007, p.2) particularly mathematics. He insists that too many students lose faith in their own intelligence, and too much effort is directed at creating artificial differences between fast and slow, gifted and “special”, advanced and delayed.

And worse yet, procedural approaches to the teaching of mathematics that create problems of understanding and engagement are applied with even more vigor in remedial programs designed to help those very students for whom such practices did not work in the first place.

A growing number of researchers argue that other approaches are needed to help students learn mathematics. “Today, mathematics education faces two major challenges: raising the floor by expanding achievement for all, and lifting the ceiling of achievement to better prepare future leaders in mathematics, as well as in science, engineering, and technology. At first glance, these appear to be mutually exclusive” (Research Points, 2006, p.1). But are they? Is it possible to design learning that engages the vast majority of students in higher mathematics learning?

To answer these questions, I designed a research study to determine whether the principles of Universal Design for Learning (UDL) resulted in increased student mathematical proficiency and achievement for all students in a typical Grade 7 classroom. Was it possible, in a regular classroom to lift the ceiling and raise the floor?

Building Math Proficiency

Mathematical proficiency is developed through five interwoven and interdependent strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Mathematical proficiency cannot be achieved by focusing on only one or two strands (Kilpatrick, et.al., 2001).

The Grade 7 students in this research study undertook a study of the concepts related to geometry in particular shapes and solids. They worked with the idea that two- and three-dimensional objects with or without curved surfaces can be described, classified, and analyzed by their attributes.

Examples of Mathematical Understandings:

- Point, line, line segment, and plane are the core attributes of space objects, and real-world situations can be used to think about these attributes.
- Polygons can be described uniquely by their sides and angles.
- Polygons can be constructed from or decomposed into other polygons.
- Triangles and quadrilaterals can be described, categorized, and named based on the relative lengths of their sides and the sizes of their angles.
- All polyhedra can be described completely by their faces, edges, and vertices.
- Some shapes or combinations of shapes can be put together without overlapping to completely cover the plane.
- There is more than one way to classify most shapes and solids.

Guided by the definition of mathematical proficiency and principles of how people learn derived from the learning sciences, I designed interventions in teaching concepts of geometry to a class of Grade 7 students of mixed ability as determined by Individual Program Plans (IPP's) in place at the time of the research. The design of the intervention was based on the following assumptions about effective mathematics instruction:

- connections between and among the proficiency strands are inherent;

- every student can make progress along every math curricular strand;
- mathematical reasoning as a set of practices and norms is (1) collective, not merely individual or idiosyncratic, and (2) rooted in the discipline of mathematics
- teaching students to reason mathematically requires them to formulate conjectures, create and use networks of concepts and processes, justify solutions and find proof that their solutions are valid;
- lessons are structured to address misconceptions, build coherence and help students make connections that are not inherently obvious; and
- effective instructional design integrates a sequence of instructional activities into a coherent whole. Students need coherent mental representations in order to use those representations to form new knowledge

In the past, this type of math instruction was generally reserved for students thought to be innately talented in mathematics. My interest was to test the hypothesis that interventions in instructional design planning and teaching would permit all students in an ordinary classroom to build and demonstrate mathematical proficiencies.

Designing Math Learning For All

Three elements converged in decisions about the research intervention designed for this study: a clear definition of mathematical proficiency; alignment of that definition with findings from the learning sciences; and key assumptions about the teaching of mathematics. A fourth element was required in order to specifically address (1) students with identified learning needs and (2) the role of technology in the mathematics classroom.

Recent educational innovations, such as differentiated instruction and universal design for learning (UDL), offer insights into proactively planning instruction that embraces academic diversity characteristic of most ordinary classrooms. UDL is grounded in emerging insights about brain development, learning, and digital media. Researchers, Rose, Meyer and Hitchcock (2005) observed that the disconnect between an increasingly diverse student population and our current “one-size-fits-all” curriculum will not produce the desired academic achievement gains expected in the 21st century. Drawing on the historical application of universal design in architecture, they advance UDL as a means of focusing educational research, development, and practice on understanding diversity, technology, and learning.

Because of the structure of our current education system, which makes sharp distinctions between “regular” and “special needs coded” students, UDL has been taken up most seriously in special education where issues of access to high quality learning experiences for variously identified special needs students carry a particular urgency. But all proponents of UDL actually make much larger claims for their ideas:

- that diversity in the classroom is the norm rather than a problem to be fixed;
- that paying attention to what does—and does not—work for students generally relegated to the margins will improve learning for all;
- that all students can meet similar learning goals if curricula, instruction and assessment are radically reconceptualized; and
- that effective use of technology enables all students to represent, express and engage with ideas in multiple ways not generally seen in conventional classrooms.

To the extent that principles of UDL are increasingly familiar, I will not summarize the field in general. Rather, I draw on the following key principles as they provide a focus for dismantling procedural methods of teaching mathematics in favor of developing mathematical proficiency for all.

1. Procedural approaches to the teaching of mathematics privilege naked independence (Edyburn 2006, p.22): the notion that completing tasks without performance-enhancing access to technology is superior to performance that is enhanced through technology. This out-dated formulation of what it means to be an educated person ensures that academic achievement is reserved only for able-bodied individuals, and only for those individuals who are able to succeed without external support, resources or technology. For many students, “technology can be the difference between students with special needs sitting in a classroom watching others participate and all students participating fully” (Bausch and Hasselbring, 2005, p.9). And for all, access to a wide variety of digital media permits mathematical explorations that are difficult or impossible with only pencil and paper. These media include (but are not limited to) spreadsheets and databases, simulations, dynamic geometry, programming, interactive games, etc. Accessible classrooms are media rich (Friesen 2006).
2. Disability can be conceived as a mismatch between the learner’s needs and the education offered (Rothberg and Treviranus 2006). Rather than conceptualized as a personal trait, disability can be seen as an artifact of the way children are taught. Many students come to dislike mathematics because of their experiences in school. Even worse, they lose faith in themselves as learners. That is, the mismatch between what is offered and what they need in order to become engaged, enthusiastic and proficient learners may actually create what come to be identified as mathematical learning disabilities.
3. UDL design principles focus on creating clear goals, flexible methods and materials, and embedded assessments that enable all learners including those with disabilities to access knowledge, participate and progress.
4. Learning is about deep understanding, constructing knowledge and developing skills and thus requires a careful balance of support, challenge and opportunity.

5. UDL calls for:

- *Multiple means of representation*, to give learners various ways of acquiring information and knowledge,
- *Multiple means of expression*, to provide learners alternatives for demonstrating what they know,
- *Multiple means of engagement*, to tap into learners' interests, offer appropriate challenges, and increase motivation

(CAST, 2006)

Conventionally, when students have difficulties learning concepts and skills outlined in the Programs of Study, modifications to instructional planning, strategies, pedagogy, resources and support are provided. Generally, material thought to be too complex or difficult for them to master is broken down into smaller, fragmented pieces. Often these are given work that appears easier, or that requires increased repetitive practice and skill-building drill on these fragmented pieces, sometimes with the aid of classroom assistants and pullout programs¹.

The prevailing assumption is that mathematical difficulties lie in the inherent inability of the individual student to master the outcomes in the mandated Program of Study in ways that are unproblematic for normal or regular students. That is the problem lies in the student not in the way the math idea is presented to the student. Accommodations are provided to remediate these difficulties to the extent possible for each disabled learner, however disability is defined for that individual. In a nutshell, differences among and between learners are generally regarded as a problem to be remedied with modification to existing programs.

Too many students do not “get” conventional approaches to the teaching of mathematics. This is not limited to those with identified learning disabilities but also includes other students struggling with the culture, language and access to learning. And it also includes those who just simply come to dislike math.

Building Assessment Into The Learning

Dynamic assessment must be intentionally built into the instructional design of student learning from the beginning. This provides learning scaffolds and feedback to the student and to the teacher. Multiple means of expression and assessment enable both teachers and students to assess what they currently know and to identify and plan the required next steps. Dynamic assessment

¹ I am not saying that practice and skill building is not necessary when learning mathematics—they are. Rather when students lose sight of how and what they are practicing relates to the bigger math idea then reasoning and understanding is forfeited.

aligns closely with teaching goals and methods. Assessment *for* learning is an integral part of the instructional design process. It was also a key aspect of the design of this research study. Constant assessment of students' developing proficiencies and their misconceptions guided the direction of the intervention on an on-going basis.

If assessment practices remain a one-size-fits-all method of sorting out ability hierarchies in the classroom, rather than guide learning and instruction, they will give incomplete pictures of the multiple ways in which individuals develop their proficiencies. Well-designed, embedded, dynamic assessment practices have the potential to remove many of the current barriers to learning in the mathematics classroom.

In designing the pedagogical interactions in this study, I placed a strong emphasis on assessment for learning, defined as

any assessment for which the first priority in its design and practice is to serve the purpose of promoting pupils' learning. It thus differs from assessment designed primarily to serve the purposes of accountability, or of ranking, or of certifying competence. An assessment activity can help learning if it provides information to be used as feedback, by teachers, and by their pupils in assessing themselves and each other, to modify the teaching and learning activities in which they are engaged. Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching work to meet learning needs. (Black, et al. 2002, p.2-3).

Assessment for learning involved:

- Building criteria for success with the students.
- Ongoing sustained dialogue with students. I spent a great deal of effort went into examining the geometric mathematical territory so the teacher and I could engage students in dialogue around questions that were worthy mathematically and which could assist students in making connections, developing reasoning and building mathematical proficiency.
- An analytic trait rubric was designed, thoroughly discussed and made available to the students before the study started. The students used it constantly throughout the study as a roadmap to success. Using both the criteria that we had collaboratively established and the rubric, the students always knew where they were, where they were going and what they needed to do address the gap. From this rubric, students found proof of learning in their own work and set goals for next learning steps.
- Specific daily constructive feedback, both written and oral from the teacher and me, and from each other around identified criteria.

- Feedback from the dynamic geometry application. Students got immediate feedback from the dynamic geometry application when a construction wasn't working as they had intended it to.

Dislodging A Teaching Script

The students in this study represented a fairly homogeneous population in terms of their demographic background. However, the number of students in this class with identified disabilities was 24.7% above the provincial average.

Mrs. Jamieson, the teacher, had more than ten years of teaching experience. She volunteered to participate in this study because she saw it as an opportunity to learn. She was an extremely conscientious teacher who cared deeply that her students not repeat her own school math experience. However, she noted that she relied heavily on the math textbook and worksheets to guide the content of her lessons. In describing the ways in which she taught the content she stated:

I typically talk to them about have they done it before, what do you know about it, here is an example, now you do one and then it's a worksheet or something out of a textbook.

What Mrs. Jamieson has just described is a lesson pattern that has been well documented by Stigler and Hiebert (1999). First identified in the 1995 International Association for the Evaluation of Education Achievement (IEA) Third International Mathematics and Science Videotape Study (TIMSS), this lesson pattern is known as the American teaching script. The lesson pattern repeats in the following way:

- Review previous material
- Demonstrate how to solve problems for the day
- Practice the demonstrated problem
- Correct seatwork and assign homework

In fact, so ingrained was this teaching script, that when teachers started to work with more robust, complex problems in the years after the 1995 TIMSS study, they transformed these “problems designed for teaching rich mathematical concepts into routine procedural exercises”(U.S. Department of Education, 2004).

The pervasiveness of the North American script for teaching mathematics provides essential context as we start to focus on the potential for UDL in the mathematics classroom. If fundamental principles of UDL become tied to a teaching script focused on practicing routine, procedural exercises, with little to no dynamic assessment, then mathematical proficiency for all students will not become a reality. Designing mathematics learning for and teaching mathematics in a UDL classroom suddenly get far more complicated than first imagined, for

now it is not just the principles of UDL and assessment for learning that need to be brought to bear, but also what research tells us about gaining mathematical proficiency.

This point is of crucial importance in designing instruction so all students learn. If the ways in which mathematics is taught involves practicing routine procedures rather than building mathematical proficiency, then it matters little that teachers represent this information in multiple ways or that students have the opportunity to express these routine procedures in multiple ways. The mathematics teaching remains fundamentally problematic.

To relieve the monotony symptomatic of this routine teaching script, the educational community in North America has attempted to make math more fun.

You'll find one attempt after another, especially recently, to lure children into math by making it fun. 'Manipulables' replace memorizing times tables; a pattern is discovered, and then another; shapes are folded; bells run; numbers dance. All this has the welcome effect of not giving fear and loathing even a look-in: games become the arena where minds encounter math. The problem usually is, however, that these encounters stay superficial—a decorative rather than an architectural instinct is catered to (Kaplan and Kaplan, 2007, p.131).

This is not to say that there is no place for manipulatives in the mathematics classroom. Rather, it is to insist that the mathematics is not contained in the manipulatives, themselves. Mathematics itself is the study of connections: how things ideally must and, in fact, do sort together—beyond, around, and within us.

A recent longitudinal design-based research study by Swain and Swan (2007) reported that

some teachers believed that the approaches were about 'standing back and letting the learners discover things for themselves'. ... Some teachers became aware of the shortcomings of transmission methods of teaching and recognised that 'telling' was not always an effective way of helping learners to understand concepts. Perhaps in reaction to this, they moved to an extreme position of 'not telling' (p.32 – 33).

A teacher working with the principles of UDL in the classroom has the ability to represent any mathematical concept in multiple ways. It is the conceptual understanding, which involves an understanding of concepts, operations and relations, that is focus of the instruction, not the number and types of manipulatives. It is all too easy to slip into decorative forms of mathematics in the name of fun as alternatives to routine procedures. When this happens, the students are frequently abandoned, left to "discover" the mathematics for themselves. The result: students end up with the same lack of mathematical proficiency as those who formerly received an endless repetition of routine procedures.

Teaching for mathematical proficiency requires that the teachers design a learning environment that provides a solid foundation of detailed knowledge and clarity about the core concepts around which that knowledge is organized to support effective learning. The type of practice required to promote mathematical proficiency stands in sharp contrast both to transmission-type pedagogies and to discovery-type pedagogies. Rather, the type of practice that builds mathematical proficiency requires that students be brought into a place of deep understanding through a collaborative relationship between different information and facts students are learning, between the procedures they are learning and the underlying concepts, through robust, rich problems and investigations that reach deep into fundamental mathematical ideas.

Upsetting The Natural Order

Towards the end of the second week of the study it became increasingly difficult to identify the nine students with identified learning disabilities. They accessed the same learning curriculum, were given the choice and support to express their learning using methods of their choice and they were not isolated from their peers.

Many of the intriguing questions during our classroom conversations came from the students with identified learning difficulties. Concepts such as Pi, circumference, area and perimeter intrigued them. They questioned with confidence, added their thoughts to the general discussions, and became active participants in their own learning. That is, students who typically had difficulty understanding mathematics, those for whom mathematics typically didn't make much sense, started to speak up.

What I did not anticipate was the extent to which their increasing proficiency temporarily bothered some of the students who saw themselves, or were seen by others as being good at math. They protested: *"Hey, how do you know that?"* *"You don't get math."* Then in quiet, hushed conversations, out of what they thought were the ears of their teacher and me, their complaints grew.

It wasn't fair that *those* kids were getting math. Fortunately, this attitude changed as they realized that learning in this way was not a competition. Too often, when marks are used as a sorting device, achievement becomes a zero sum game: the advance of some is gained at the expense of others. As students' engagement increased, they lost this fear.

Initially, the academic order of the classroom was disrupted, a social order created by conventional educational structures and processes and their organizing principles and assumptions. It was beyond the scope of this study to determine whether the reactions of these students would be in any way typical of other classrooms when those who "everyone knows" are left behind suddenly emerge as equally able and engaged. For now, however, we are comfortable noting two things.

First, “equality is—and must continue to be—a key goal of any public education system”; however, “we need new ways of thinking about equality, ways that do not involve sameness, or one-size-fits-all approaches” (Gilbert, 2005, p.102). And it would appear that one of those new ways of thinking may involve re-interpreting the unintended consequences of meeting individual needs by identifying some students as inherently less able than others when, in fact, their perceived disabilities are to some extent artifacts of our own structures and pedagogies.

Second, it may be important to examine the history of mathematics as a gatekeeper subject that has traditionally been used to separate the academically able sheep from the less talented goats. Some members of the community of mathematics educators and researchers have been working to expose what they call the myth of mathematical talent as the primary way to explain why some students “get” math and others do not.

This study revealed that in a short time, four weeks, even in a classroom with a disproportionately large number of identified special needs students, it is possible to raise both ceiling and floor simultaneously. All students showed statistically significant improvement in achievement. All students made gains in all five strands of mathematical proficiency. All students engaged with and understood difficult mathematical ideas when they were provided with worthy mathematics, dynamic formative assessment to guide their learning and the teacher’s teaching and deep, probing conversations that required students to justify their solutions and thinking.

Choosing to do so becomes, then, a matter of policy. How badly do we want equality for all students, and are we prepared to weather what may be inevitable storms as we start to shake up the status quo from the highest levels right down to the playground?

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